



International Baccalaureate®
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Diploma Programme

Mathematics: **PRE IB - GRADE 10** **formula booklet**

For use during the course and in the examinations
First examinations 2021

Version 1.1

**DO NOT WRITE ANY NOTES ON THIS
FORMULA BOOKLET**

Notation list

SL and HL

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, \dots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \dots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$n(A)$	the number of elements in the finite set A
$\{x \mid \quad\}$	the set of all x such that
$\in$	is an element of
$\notin$	is not an element of
$\emptyset$	the empty (null) set
U	the universal set
$\cup$	union
$\cap$	intersection
A'	the complement of the set A
$a^{1/2}, \sqrt{a}$	a to the power $\frac{1}{2}$, square root of a (if $a \geq 0$ then $\sqrt{a} \geq 0$)
$a^{1/n}, \sqrt[n]{a}$	a to the power of $\frac{1}{n}$, n^{th} root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$a^{-n} = \frac{1}{a^n}$	a to the power of $-n$, reciprocal of a^n
$ x $	the modulus or absolute value of x , that is $\begin{cases} x & \text{for } x \geq 0, x \in \mathbb{R} \\ -x & \text{for } x < 0, x \in \mathbb{R} \end{cases}$

Prior learning – SL and HL

Area of a parallelogram	$A = bh$, where b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b is the base, h is the height
Area of a trapezoid	$A = \frac{1}{2}(a + b)h$, where a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Volume of a cuboid	$V = lwh$, where l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$, where r is the radius, h is the height
Volume of prism	$V = Ah$, where A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi rh$, where r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Solutions of a quadratic equation	The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$
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Topic I: Number and algebra – SL and HL

SL 1.2	The n th term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d); S_n = \frac{n}{2}(u_1 + u_n)$
SL 1.3	The n th term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$
SL 1.4	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$, where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest
SL 1.5	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$, where $a > 0, b > 0, a \neq 1$
SL 1.6	Percentage error	$\mathcal{E} = \left \frac{v_A - v_E}{v_E} \right \times 100\%$, where v_E is the exact value and v_A is the approximate value of v

Topic I: Number and algebra – HL only

AHL 1.9	Laws of logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ for $a, x, y > 0$
AHL 1.11	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}, r < 1$
SL 1.9	Binomial theorem $n \in \mathbb{N}$	$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$ ${}^nC_r = \frac{n!}{r!(n-r)!}$

AHL 1.12	Complex numbers	$z = a + bi$
	Discriminant	$\Delta = b^2 - 4ac$
AHL 1.13	Modulus-argument (polar) and exponential (Euler) form	$z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$
AHL 1.14	De Moivre's theorem	$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$

Topic 2: Functions – SL and HL

SL 2.1	Equations of a straight line	$y = mx + c; ax + by + d = 0; y - y_1 = m(x - x_1)$
	Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
SL 2.5	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
SL 2.9	Exponential and logarithmic functions	$a^x = e^{x \ln a}; \log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$

Topic 2: Functions – HL only

Remainder Theorem

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

Factor Theorem

c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

Fundamental Theorem of Algebra

Every polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad (n \geq 1, a_n \neq 0)$$

with complex coefficients has at least one complex zero.

SUM AND PRODUCT OF ROOTS THEOREM

For the polynomial equation of degree n given by $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$,

AHL 2.12	Sum and product of the roots of polynomial equations of the form $\sum_{r=0}^n a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$
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Topic 3: Geometry and trigonometry – SL and HL

SL 3.1	Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2) Volume of a right-pyramid Volume of a right cone Area of the curved surface of a cone Volume of a sphere Surface area of a sphere	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ $V = \frac{1}{3} Ah$, where A is the area of the base, h is the height $V = \frac{1}{3} \pi r^2 h$, where r is the radius, h is the height $A = \pi r l$, where r is the radius, l is the slant height $V = \frac{4}{3} \pi r^3$, where r is the radius $A = 4\pi r^2$, where r is the radius
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SL 3.2	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2} ab \sin C$
SL 3.4	Length of an arc	$l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle measured in degrees, r is the radius
	Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$, where θ is the angle measured in degrees, r is the radius

SL 3.5	Identity for $\tan \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
SL 3.6	Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$
	Double angle identities	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

Topic 3: Geometry and trigonometry – HL only

AHL 3.7	Length of an arc	$l = r\theta$, where r is the radius, θ is the angle measured in radians
	Area of a sector	$A = \frac{1}{2} r^2 \theta$, where r is the radius, θ is the angle measured in radians
AHL 3.9	Reciprocal trigonometric identities	$\sec \theta = \frac{1}{\cos \theta}$ $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
	Pythagorean identities	$1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

AHL 3.10	Compound angle identities Double angle identity for tan	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
AHL 3.12	Magnitude of a vector	$ \mathbf{v} = \sqrt{v_1^2 + v_2^2 + v_3^2}, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

AHL 3.13	Scalar product Angle between two vectors	$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w} \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{v} \text{ and } \mathbf{w}$ $\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{ \mathbf{v} \mathbf{w} }$
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