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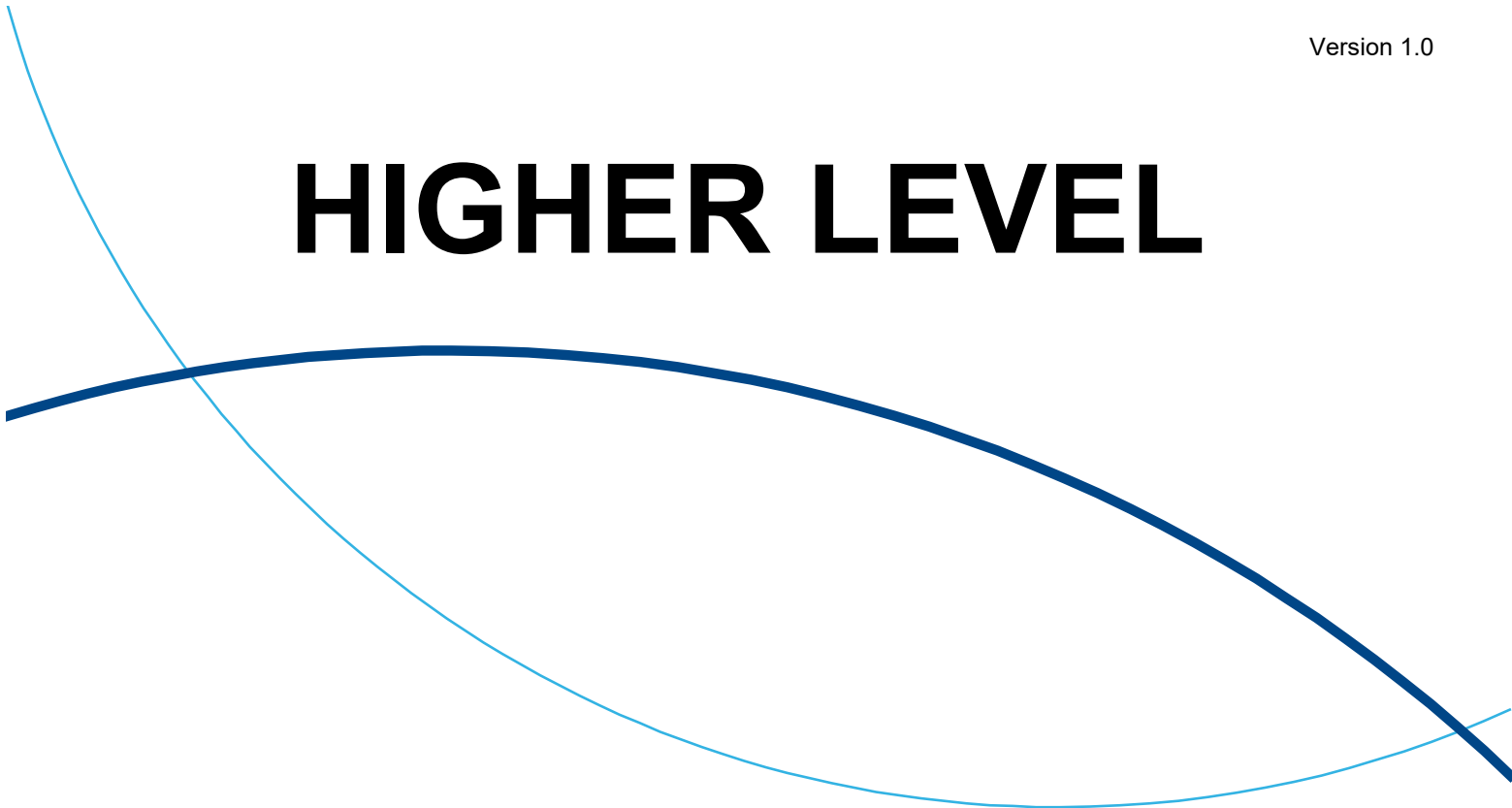
Diploma Programme

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# Mathematics: applications and interpretation HL formula booklet

For use during the course and in the examinations  
First examinations 2021

Version 1.0

Two decorative blue curves are positioned behind the 'HIGHER LEVEL' text. One is a thick dark blue curve that starts on the left, rises to a peak, and then descends towards the right. The other is a thin light blue curve that starts high on the left, dips down, and then rises again towards the right.

# HIGHER LEVEL



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## Topic 1: Number and algebra – HL

|             |                                                                                                                                |                                                                                                                                                                                                                                                                                                                           |
|-------------|--------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>1.2</b>  | <p>The <math>n</math>th term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>    | $u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d); S_n = \frac{n}{2}(u_1 + u_n)$                                                                                                                                                                                                                                     |
| <b>1.3</b>  | <p>The <math>n</math>th term of a geometric sequence</p> <p>The sum of <math>n</math> terms of a finite geometric sequence</p> | $u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$                                                                                                                                                                                                                             |
| <b>1.11</b> | The sum of an infinite geometric sequence                                                                                      | $S_\infty = \frac{u_1}{1 - r},  r  < 1$                                                                                                                                                                                                                                                                                   |
| <b>1.4</b>  | Compound interest                                                                                                              | $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ <p>where <math>FV</math> is the future value,<br/> <math>PV</math> is the present value, <math>n</math> is the number of years,<br/> <math>k</math> is the number of compounding periods per year,<br/> <math>r\%</math> is the nominal annual rate of interest</p> |
| <b>1.5</b>  | Exponents and logarithms                                                                                                       | $a^x = b \Leftrightarrow x = \log_a b, \text{ where } a > 0, b > 0, a \neq 1$                                                                                                                                                                                                                                             |
| <b>1.9</b>  | Laws of logarithms                                                                                                             | $\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ <p>for <math>a, x, y &gt; 0</math></p>                                                                                                                                                                             |
| <b>1.6</b>  | Percentage error                                                                                                               | $\mathcal{E} = \left  \frac{v_A - v_E}{v_E} \right  \times 100\%, \text{ where } v_E \text{ is the exact value and } v_A \text{ is the approximate value of } v$                                                                                                                                                          |
| <b>1.12</b> | <p>Complex numbers</p> <p>Discriminant</p>                                                                                     | $z = a + bi$ $\Delta = b^2 - 4ac$                                                                                                                                                                                                                                                                                         |
| <b>1.13</b> | Modulus-argument (polar) and exponential (Euler) form                                                                          | $z = r(\cos \theta + i \sin \theta) = re^{i\theta} = r \operatorname{cis} \theta$                                                                                                                                                                                                                                         |

|             |                                      |                                                                                                                                                     |
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| <b>1.14</b> | Determinant of a $2 \times 2$ matrix | $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A =  A  = ad - bc$                                                               |
|             | Inverse of a $2 \times 2$ matrix     | $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, ad \neq bc$ |
| <b>1.15</b> | Power formula for a matrix           | $M^n = PD^nP^{-1}$ , where $P$ is the matrix of eigenvectors and $D$ is the diagonal matrix of eigenvalues                                          |

## Topic 2: Functions – HL

### Prior learning – HL

|                                   |                                                                                             |
|-----------------------------------|---------------------------------------------------------------------------------------------|
| Solutions of a quadratic equation | The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$ |
|-----------------------------------|---------------------------------------------------------------------------------------------|

|            |                                                       |                                                                            |
|------------|-------------------------------------------------------|----------------------------------------------------------------------------|
| <b>2.1</b> | Equations of a straight line                          | $y = mx + c; ax + by + d = 0; y - y_1 = m(x - x_1)$                        |
|            | Gradient formula                                      | $m = \frac{y_2 - y_1}{x_2 - x_1}$                                          |
| <b>2.5</b> | Axis of symmetry of the graph of a quadratic function | $f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$ |
| <b>2.9</b> | Logistic function                                     | $f(x) = \frac{L}{1 + Ce^{-kx}}, L, k, C > 0$                               |

## Topic 3: Geometry and trigonometry – HL

### Prior learning – HL

|                                                                                            |                                                                                         |
|--------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| Area of a parallelogram                                                                    | $A = bh$ , where $b$ is the base, $h$ is the height                                     |
| Area of a triangle                                                                         | $A = \frac{1}{2}(bh)$ , where $b$ is the base, $h$ is the height                        |
| Area of a trapezoid                                                                        | $A = \frac{1}{2}(a + b)h$ , where $a$ and $b$ are the parallel sides, $h$ is the height |
| Area of a circle                                                                           | $A = \pi r^2$ , where $r$ is the radius                                                 |
| Circumference of a circle                                                                  | $C = 2\pi r$ , where $r$ is the radius                                                  |
| Volume of a cuboid                                                                         | $V = lwh$ , where $l$ is the length, $w$ is the width, $h$ is the height                |
| Volume of a cylinder                                                                       | $V = \pi r^2 h$ , where $r$ is the radius, $h$ is the height                            |
| Volume of prism                                                                            | $V = Ah$ , where $A$ is the area of cross-section, $h$ is the height                    |
| Area of the curved surface of a cylinder                                                   | $A = 2\pi rh$ , where $r$ is the radius, $h$ is the height                              |
| Distance between two points $(x_1, y_1)$ and $(x_2, y_2)$                                  | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$                                              |
| Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$ | $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$                               |

|     |                                                                                                      |                                                                                |
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| 3.1 | Distance between two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$                                  | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$                     |
|     | Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ | $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ |
|     | Volume of a right-pyramid                                                                            | $V = \frac{1}{3}Ah$ , where $A$ is the area of the base, $h$ is the height     |

|            |                                      |                                                                                                              |
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|            | Volume of a right cone               | $V = \frac{1}{3}\pi r^2 h$ , where $r$ is the radius, $h$ is the height                                      |
|            | Area of the curved surface of a cone | $A = \pi r l$ , where $r$ is the radius, $l$ is the slant height                                             |
|            | Volume of a sphere                   | $V = \frac{4}{3}\pi r^3$ , where $r$ is the radius                                                           |
|            | Surface area of a sphere             | $A = 4\pi r^2$ , where $r$ is the radius                                                                     |
| <b>3.2</b> | Sine rule                            | $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$                                                     |
|            | Cosine rule                          | $c^2 = a^2 + b^2 - 2ab \cos C$ ; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$                                      |
|            | Area of a triangle                   | $A = \frac{1}{2}ab \sin C$                                                                                   |
| <b>3.4</b> | Length of an arc                     | $l = \frac{\theta}{360} \times 2\pi r$ , where $\theta$ is the angle measured in degrees, $r$ is the radius  |
|            | Area of a sector                     | $A = \frac{\theta}{360} \times \pi r^2$ , where $\theta$ is the angle measured in degrees, $r$ is the radius |
| <b>3.7</b> | Length of an arc                     | $l = r\theta$ , where $r$ is the radius, $\theta$ is the angle measured in radians                           |
|            | Area of a sector                     | $A = \frac{1}{2}r^2\theta$ , where $r$ is the radius, $\theta$ is the angle measured in radians              |
| <b>3.8</b> | Identities                           | $\cos^2 \theta + \sin^2 \theta = 1$<br><br>$\tan \theta = \frac{\sin \theta}{\cos \theta}$                   |

|      |                                                                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|------|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3.9  | Transformation matrices                                                           | $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}, \text{ reflection in the line } y = (\tan \theta)x$<br>$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}, \text{ horizontal stretch / stretch parallel to } x\text{-axis with a scale factor of } k$<br>$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}, \text{ vertical stretch / stretch parallel to } y\text{-axis with a scale factor of } k$<br>$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}, \text{ enlargement, with a scale factor of } k, \text{ centre } (0, 0)$<br>$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \text{ anticlockwise/counter-clockwise rotation of angle } \theta \text{ about the origin } (\theta > 0)$<br>$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \text{ clockwise rotation of angle } \theta \text{ about the origin } (\theta > 0)$ |
| 3.10 | Magnitude of a vector                                                             | $ \mathbf{v}  = \sqrt{v_1^2 + v_2^2 + v_3^2}, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| 3.11 | <p>Vector equation of a line</p> <p>Parametric form of the equation of a line</p> | $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$<br>$x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |

|      |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
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| 3.13 |  | <p>Scalar product</p> $\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\mathbf{v} \cdot \mathbf{w} =  \mathbf{v}   \mathbf{w}  \cos \theta, \text{ where } \theta \text{ is the angle between } \mathbf{v} \text{ and } \mathbf{w}$ <p>Angle between two vectors</p> $\cos \theta = \frac{v_1 w_1 + v_2 w_2 + v_3 w_3}{ \mathbf{v}   \mathbf{w} }$ <p>Vector product</p> $\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}, \text{ where } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $ \mathbf{v} \times \mathbf{w}  =  \mathbf{v}   \mathbf{w}  \sin \theta, \text{ where } \theta \text{ is the angle between } \mathbf{v} \text{ and } \mathbf{w}$ <p>Area of a parallelogram</p> $A =  \mathbf{v} \times \mathbf{w}  \text{ where } \mathbf{v} \text{ and } \mathbf{w} \text{ form two adjacent sides of a parallelogram}$ |
|------|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## Topic 4: Statistics and probability – HL

|            |                                                  |                                                                           |
|------------|--------------------------------------------------|---------------------------------------------------------------------------|
| <b>4.2</b> | Interquartile range                              | $\text{IQR} = Q_3 - Q_1$                                                  |
| <b>4.3</b> | Mean, $\bar{x}$ , of a set of data               | $\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$ , where $n = \sum_{i=1}^k f_i$ |
| <b>4.5</b> | Probability of an event $A$                      | $P(A) = \frac{n(A)}{n(U)}$                                                |
|            | Complementary events                             | $P(A) + P(A') = 1$                                                        |
| <b>4.6</b> | Combined events                                  | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$                                 |
|            | Mutually exclusive events                        | $P(A \cup B) = P(A) + P(B)$                                               |
|            | Conditional probability                          | $P(A B) = \frac{P(A \cap B)}{P(B)}$                                       |
|            | Independent events                               | $P(A \cap B) = P(A) P(B)$                                                 |
| <b>4.7</b> | Expected value of a discrete random variable $X$ | $E(X) = \sum_{i=1}^k x_i P(X = x_i)$                                      |
| <b>4.8</b> | Binomial distribution<br>$X \sim B(n, p)$        |                                                                           |
|            | Mean                                             | $E(X) = np$                                                               |
|            | Variance                                         | $\text{Var}(X) = np(1 - p)$                                               |



## Topic 5: Calculus – HL

|            |                        |                                                                                                                                   |
|------------|------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| <b>5.3</b> | Derivative of $x^n$    | $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$                                                                                         |
| <b>5.9</b> | Derivative of $\sin x$ | $f(x) = \sin x \Rightarrow f'(x) = \cos x$                                                                                        |
|            | Derivative of $\cos x$ | $f(x) = \cos x \Rightarrow f'(x) = -\sin x$                                                                                       |
|            | Derivative of $\tan x$ | $f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$                                                                            |
|            | Derivative of $e^x$    | $f(x) = e^x \Rightarrow f'(x) = e^x$                                                                                              |
|            | Derivative of $\ln x$  | $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$                                                                                    |
|            | Chain rule             | $y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$                                |
|            | Product rule           | $y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$                                                            |
|            | Quotient rule          | $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$                                       |
| <b>5.5</b> | Integral of $x^n$      | $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$                                                                          |
| <b>5.8</b> | The trapezoidal rule   | $\int_a^b y dx \approx \frac{1}{2} h ((y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})),$ <p>where <math>h = \frac{b-a}{n}</math></p> |

|             |                                                                                                                                                             |                                                                                                                                                                                                                                                                                               |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>5.11</b> | Standard integrals                                                                                                                                          | $\int \frac{1}{x} dx = \ln x  + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int \frac{1}{\cos^2 x} dx = \tan x + C$ $\int e^x dx = e^x + C$                                                                                                                             |
| <b>5.12</b> | <p>Area of region enclosed by a curve and <math>x</math> or <math>y</math>-axes</p> <p>Volume of revolution about <math>x</math> or <math>y</math>-axes</p> | $A = \int_a^b  y  dx \text{ or } A = \int_a^b  x  dy$ $V = \int_a^b \pi y^2 dx \text{ or } V = \int_a^b \pi x^2 dy$                                                                                                                                                                           |
| <b>5.13</b> | <p>Acceleration</p> <p>Distance travelled from <math>t_1</math> to <math>t_2</math></p> <p>Displacement from <math>t_1</math> to <math>t_2</math></p>       | $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \frac{dv}{ds}$ $\text{distance} = \int_{t_1}^{t_2}  v(t)  dt$ $\text{displacement} = \int_{t_1}^{t_2} v(t) dt$                                                                                                                                     |
| <b>5.16</b> | <p>Euler's method</p> <p>Euler's method for coupled systems</p>                                                                                             | $y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h, \text{ where } h \text{ is a constant (step length)}$ $x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n)$ $y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$ $t_{n+1} = t_n + h$ <p>where <math>h</math> is a constant (step length)</p> |
| <b>5.17</b> | Exact solution for coupled linear differential equations                                                                                                    | $\mathbf{x} = A e^{\lambda_1 t} \mathbf{p}_1 + B e^{\lambda_2 t} \mathbf{p}_2$                                                                                                                                                                                                                |